C4 VECTORS — Worksheet B

1 The points A, B and C have coordinates (6, 1), (2, 3) and (–4, 3) respectively and O is the origin. Find, in terms of **i** and **j**, the vectors

a $\overrightarrow{OA}$ **b** $\overrightarrow{AB}$ **c** $\overrightarrow{BC}$ **d** $\overrightarrow{CA}$

2 Given that $\mathbf{p} = \mathbf{i} - 3\mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} + 2\mathbf{j}$, find expressions in terms of **i** and **j** for

a $4\mathbf{p}$ **b** $\mathbf{q} - \mathbf{p}$ **c** $2\mathbf{p} + 3\mathbf{q}$ **d** $4\mathbf{p} - 2\mathbf{q}$

3 Given that $\mathbf{p} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, find

a $|\mathbf{p}|$ **b** $|2\mathbf{q}|$ **c** $|\mathbf{p} + 2\mathbf{q}|$ **d** $|3\mathbf{q} - 2\mathbf{p}|$

4 Given that $\mathbf{p} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{q} = \mathbf{i} - 3\mathbf{j}$, find, in degrees to 1 decimal place, the angle made with the vector **i** by the vector

a $\mathbf{p}$ **b** $\mathbf{q}$ **c** $5\mathbf{p} + \mathbf{q}$ **d** $\mathbf{p} - 3\mathbf{q}$

5 Find a unit vector in the direction

a $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$ **c** $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

6 Find a vector

a of magnitude 26 in the direction $5\mathbf{i} + 12\mathbf{j}$,

b of magnitude 15 in the direction $-6\mathbf{i} - 8\mathbf{j}$,

c of magnitude 5 in the direction $2\mathbf{i} - 4\mathbf{j}$.

7 Given that $\mathbf{m} = 2\mathbf{i} + \lambda\mathbf{j}$ and $\mathbf{n} = \mu\mathbf{i} - 5\mathbf{j}$, find the values of λ and μ such that

a $\mathbf{m} + \mathbf{n} = 3\mathbf{i} - \mathbf{j}$ **b** $2\mathbf{m} - \mathbf{n} = -3\mathbf{i} + 8\mathbf{j}$

8 Given that $\mathbf{r} = 6\mathbf{i} + c\mathbf{j}$, where c is a positive constant, find the value of c such that

a $\mathbf{r}$ is parallel to the vector $2\mathbf{i} + \mathbf{j}$ **b** $\mathbf{r}$ is parallel to the vector $-9\mathbf{i} - 6\mathbf{j}$

c $|\mathbf{r}| = 10$ **d** $|\mathbf{r}| = 3\sqrt{5}$

9 Given that $\mathbf{p} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} - 2\mathbf{j}$,

a find the values of a and b such that $a\mathbf{p} + b\mathbf{q} = -5\mathbf{i} + 13\mathbf{j}$,

b find the value of c such that $c\mathbf{p} + \mathbf{q}$ is parallel to the vector $\mathbf{j}$,

c find the value of d such that $\mathbf{p} + d\mathbf{q}$ is parallel to the vector $3\mathbf{i} - \mathbf{j}$.

10 Relative to a fixed origin O, the points A and B have position vectors $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$ respectively. Find

a the vector $\overrightarrow{AB}$,

b $|\overrightarrow{AB}|$,

c the position vector of the mid-point of AB,

d the position vector of the point C such that $OABC$ is a parallelogram.

11 Given the coordinates of the points A and B, find the length AB in each case.

a $A\,(4, 0, 9)$, $B\,(2, -3, 3)$ **b** $A\,(11, -3, 5)$, $B\,(7, -1, 3)$

12 Find the magnitude of each vector.

a $4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ **b** $\mathbf{i} + \mathbf{j} + \mathbf{k}$ **c** $-8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ **d** $3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$

13 Find

a a unit vector in the direction $5\mathbf{i} - 2\mathbf{j} + 14\mathbf{k}$,

b a vector of magnitude 10 in the direction $2\mathbf{i} + 11\mathbf{j} - 10\mathbf{k}$,

c a vector of magnitude 20 in the direction $-5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

14 Given that $\mathbf{r} = \lambda\mathbf{i} + 12\mathbf{j} - 4\mathbf{k}$, find the two possible values of λ such that $|\mathbf{r}| = 14$.

15 Given that $\mathbf{p} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -2 \\ 5 \\ -3 \end{pmatrix}$, find as column vectors,

a $\mathbf{p} + 2\mathbf{q}$ **b** $\mathbf{p} - \mathbf{r}$ **c** $\mathbf{p} + \mathbf{q} + \mathbf{r}$ **d** $2\mathbf{p} - 3\mathbf{q} + \mathbf{r}$

16 Given that $\mathbf{r} = -2\mathbf{i} + \lambda\mathbf{j} + \mu\mathbf{k}$, find the values of λ and μ such that

a $\mathbf{r}$ is parallel to $4\mathbf{i} + 2\mathbf{j} - 8\mathbf{k}$ **b** $\mathbf{r}$ is parallel to $-5\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$

17 Given that $\mathbf{p} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$, $\mathbf{q} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{r} = 2\mathbf{i} - 4\mathbf{j} - 7\mathbf{k}$,

a find $|2\mathbf{p} - \mathbf{q}|$,

b find the value of k such that $\mathbf{p} + k\mathbf{q}$ is parallel to $\mathbf{r}$.

18 Relative to a fixed origin O, the points A, B and C have position vectors $(-2\mathbf{i} + 7\mathbf{j} + 4\mathbf{k})$, $(-4\mathbf{i} + \mathbf{j} + 8\mathbf{k})$ and $(6\mathbf{i} - 5\mathbf{j})$ respectively.

a Find the position vector of the mid-point of AB.

b Find the position vector of the point D on AC such that $AD : DC = 3 : 1$

19 Given that $\mathbf{r} = \lambda\mathbf{i} - 2\lambda\mathbf{j} + \mu\mathbf{k}$, and that $\mathbf{r}$ is parallel to the vector $(2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k})$,

a show that $3\lambda + 2\mu = 0$.

Given also that $|\mathbf{r}| = 2\sqrt{29}$ and that $\mu > 0$,

b find the values of λ and μ.

20 Relative to a fixed origin O, the points A, B and C have position vectors $\begin{pmatrix} 6 \\ -2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 12 \\ -7 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 1 \\ -8 \end{pmatrix}$ respectively.

a Find the position vector of the point M, the mid-point of BC.

b Show that O, A and M are collinear.

21 The position vector of a model aircraft at time t seconds is $(9 - t)\mathbf{i} + (1 + 2t)\mathbf{j} + (5 - t)\mathbf{k}$, relative to a fixed origin O. One unit on each coordinate axis represents 1 metre.

a Find an expression for d^2 in terms of t, where d metres is the distance of the aircraft from O.

b Find the value of t when the aircraft is closest to O and hence, the least distance of the aircraft from O.